On the isoperimetric problem with perimeter density r^p

Gyula Csató
Universidad de Concepción

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Abstract

The standard isoperimetric inequality states that among all sets with a given fixed volume (or area in dimension 2) the ball has the smallest perimeter. That is, written here for simplicity in dimension 2, the following infimum is attained by the ball

$$2\pi R = \inf\left\{\int_{\partial\Omega} 1\, d\sigma(x) : \Omega \subset \mathbb{R}^2 \text{ and } \int_{\Omega} 1\, dx = \pi R^2\right\}.$$

The isoperimetric problem with density is a generalization of this question: given two positive functions $f, g : \mathbb{R}^2 \to \mathbb{R}$ one studies the existence of minimizers of

$$I(C) = \inf\left\{\int_{\partial\Omega} g(x) d\sigma(x) : \Omega \subset \mathbb{R}^2 \text{ and } \int_{\Omega} f(x) dx = C\right\}.$$

I will first give a general overview on this type of problem. Much attention has been dedicated to the situation when $f = g = e^{\psi}$ is strictly positive and radial, which led to the log-convex density conjecture and has been solved recently. However, I will mainly talk about the situation when $f(x) = |x|^q$ and $g(x) = |x|^p$. This is a rich problem with strong variations in difficulties depending on the values of p and q. Some cases are still an open problem. Finally I will present some of my own results dealing with the case $f = 1$ and $g(x) = |x|^p$ appearing in the following references:

– Csató G., An isoperimetric problem with density and the Hardy-Sobolev inequality in $\mathbb{R}^2$, *Differential Integral Equations*, **28**, Number 9/10 (2015), 971–988.

– Csató G., On the isoperimetric problem with perimeter density r^p, submitted paper.